Quantum Many Body Physics with Strongly Interacting Matter and Light

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Interacting Light and Matter
Far From equilibrium

- **Condensed Matter**
  - Light used to probe phases or (more recently) to "induce" phases of matter

- **Ultra-Cold Atoms**
  - Light used to trap (optical lattices) to probe or to excite atoms

- **Quantum Many Body Physics with "Light" (?)**

Non-Linear Optics with Single Photons!
Outline

- Route to Non-Linear Optics with Single Photons: Coupling Light and Matter at the Quantum Level
- Many Body Physics with Atoms and Photons: Perspectives and Challenges
- Photon Lattice with Rabi Non-Linearity
Exploring
Light-Matter Interaction at the Quantum Level
Cavity Quantum Electrodynamics

Trapping photons and atoms into cavities!

J. M. Raimond, M. Brune and S. Haroche, Rev. Mod. Phys. 73, 565 (2001)

Optical/Microwave 3D cavities supporting discrete modes

Alkali or Rydberg Atoms trapped into the cavity

Light-Matter coupling (Dipole) $H_{int} \sim d \cdot E$
A Solid-State Analogue: Circuit QED

Circuit QED

✅ Microwave Transmission Line Resonators
   - Superconducting-based to avoid dissipation

✅ Artificial Atoms: Superconducting Qubits
   - Mesoscopic SC system with “atom-like” spectrum

✅ Capacitive/Inductive Coupling
How Strong Can the Coupling Between Light and Matter Be?

✓ “Poor-Man” Estimate of Light-Matter Coupling $g$

$$g = d_{at} \times \frac{E_0}{\hbar}$$

$$d_{at} \sim eL$$

$$\frac{\hbar \omega_r}{4} \sim \frac{\varepsilon_0}{2} E_0^2 V$$

✓ Fine Structure Constant Limit

M. Devoret, S. Girvin & R. Schoelkopf,
Ann Phys 16 767 (2007)

✓ Circuit QED works in a regime where $L \sim r!!$

✓ Coupling via current (i.e. magnetic field) can be even larger!
Energy Scales in CQED

- Photon Frequency/Atom splitting $\omega_r, \omega_q$
- Light-Matter Coupling (Vacuum Rabi Splitting) $g \ll \omega_r, \omega_q$
- Photon Losses and Atomic Decay Rates (Dissipation) $\kappa, \gamma$

Strong-Coupling Regime of cQED

$g \gg \kappa, \gamma$

Typical Circuit QED:

$\omega_r \sim 10 \text{ GHz}$

$g \sim 1 \text{ GHz}$

$\kappa, \gamma \sim 500 \text{ kHz}$
Effective Photon-Photon Interactions by Light-Matter Coupling
Quantum Models of Light and Matter

The Rabi Model

\[ H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger + a) (\sigma^+ + \sigma^-) \]

- Harmonic Mode (Photon)
- Anharmonic Mode (Atom)
- Linear coupling (dipole)

Often in Rotating Wave Approximation (requires \( g \ll \min(\omega_r, \omega_q) \))

\[ H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger \sigma^- + h\omega) + g (a \sigma^+ + h\omega) \]

Jaynes, Cumming (1963)
Atom-Induced Photon Non Linearity

Elementary Light-Matter Excitations: Polaritons

\[ H_{JC} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger \sigma^- + hc) \]

\[ N_{pol} = a^\dagger a + \sigma^+ \sigma^- \]

[\[ H_{JC}, N_{pol} \] = 0]

Weakly Anharmonic Spectrum

\[ E_{n\pm} = \omega_r n + \frac{\delta}{2} \pm \sqrt{\delta^2/4 + ng^2} \]

Exp Realization with Circuit QED under driving

Photon-Blockaded Transport

Steps in the Photon Transmission through a driven cavity

Strongly Dispersive Regime: \( g \ll |\omega_r - \omega_q| \)

Effective Photon Hamiltonian

\[
H_{JC} \sim \tilde{\omega}_r a^\dagger a + U_{eff} \left(a^\dagger a \right)^2
\]

\[
U_{eff} \sim \frac{g^4}{|\omega_r - \omega_q|^3}
\]
Many Body Physics with Strongly Interacting Light and Matter
Single cQED unit coupled to Quantum Baths

Experiments in Paris (T. Kontos Lab), Princeton (J. Petta Lab), ETH (K. Ennslin Lab, A. Imamoglu Lab)

Quantum Impurity Physics with Light&Matter!
M. Schiro & K. Le Hur (in preparation)

Coupled Arrays of Resonators with SC Qubits

Experiments at Princeton! (@ A. Houck’s Lab)

J. Koch and K. Le Hur PRA 80 023811 (2009)

Arrays of Resonators and SC Qubits

Hopping between adjacent resonators/cavities

Effective Photon-Photon Interactions

Delocalized bosonic particles with an effective onsite interaction...

Jaynes-Cumming Lattice Model

\[ H_{JC}^i = \omega_r a_i^\dagger a_i + \omega_q \sigma_i^+ \sigma_i^- + g \left( a_i^\dagger \sigma_i^- + hc \right) \]

\[ H_{hopping} = \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j \]

\[ H_{JCL} = \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j + \sum_i H_{JC}^i \]

Bose-Hubbard Physics(?)
Mott to Superfluid Transition of Polaritons

- **Jaynes-Cumming Lattice Model**

\[
H_{JCL} = \sum_{ij} J_{ij} \left( a_i^\dagger a_j + h.c. \right) + \sum_i H_{JC}^i - \mu \sum_i N_{pol}^i
\]

- **J=0 -- Polaritons**

\[ [H_{JC}, N_{pol}] = 0 \]

- **J<Jc -- Mott Insulator of Polaritons**

- **J>Jc -- “Superfluid”**

\[ \langle a_i \rangle \neq 0 \]

U(1) Spontaneous Symmetry Breaking

Hartmann et al, Greentree et al, Schmidt et al, Rossini et al, Tomadin et al,

J. Koch and K. Le Hur PRA 80 023811 (2009)
The importance of being a Photon

- Unlike massive particles, Photons are not conserved in Nature. Think of QED!

- Photons have zero chemical potential!

- The density of a Photon Gas is set by drive-dissipation or interaction with matter!
Can we drive photons into steady states with non trivial quantum correlations?
Open Questions

☑ Effective Chemical Potential for Polaritons?

☑ Increase Light-Matter Coupling: effective driving
  - Counter-Rotating Terms
  - $H_{int} = H_{JC} + g \left( a^\dagger \sigma^+ + \sigma^- a \right)$
  - $g \sim \omega_r, \omega_q$
  - ‘Ultra-Strong’ Coupling Regime

☑ Balancing Drive and Dissipation
  - Critical Behavior for Driven-Dissipative Quantum Systems?
  - Non-Markovian Effects? Dissipation-Induced Interactions?
  - External Drive? Coherent vs Incoherent?
  - Rate of Entropy Production? Effective Temperature?

M.Schiro et al (In preparation)
Photon Circuit QED Lattices with Rabi Non Linearity

The Rabi Lattice Model

\[ H_{RL} = \sum_{\langle ij \rangle} J_{ij} \, a_i^\dagger \, a_j + \sum_i H_{Rabi}^i \]

\[ H_{Rabi}^i = \omega_r \, a_i^\dagger \, a_i + \omega_q \, \sigma_i^+ \, \sigma_i^- + g \left( a_i^\dagger + a_i \right) \left( \sigma_i^+ + \sigma_i^- \right) \]

- **No Chemical Potential!**
  - Light-Matter interaction fix the average number of excitations in the ground state

- Can we have a non-trivial ground-state due to an “effective” drive?

- What is the effect of counter-rotating terms on the Mott-to-Superfluid Transition?
Insights from the Rabi Model

\[ H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger + a) (\sigma^+ + \sigma^-) \]

Symmetries: Parity \( Z_2 \)

\[ \Pi = e^{i\pi N_{pol}} \]

\[ \Pi^\dagger a \Pi = -a \]

\[ \Pi^\dagger \sigma^x \Pi = -\sigma^x \]

Z\text{2} symmetry enough for an "exact" solution!

Non Trivial Ground-state Properties

\[ \delta = \omega_r - \omega_q = 0 \]

Finite Number of Excitations, Squeezed, Symmetric

\[ |\Psi_{Rabi}\rangle = \sum_{n,\alpha=\pm} c_n^\alpha |n\alpha\rangle \]
A Low Energy Doublet at Strong Coupling

\[ H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger + a) (\sigma^+ + \sigma^-) \]

\( \delta = \omega_r - \omega_q = 0 \)

- Exponentially Small Splitting!
  \( \Delta = \omega_r e^{-2(g/\omega_r)^2} \)

- Ground-state has always the same (even) parity
- First excited state, with opposite parity, is almost degenerate for \( g \gg \omega_r \)
What happens at finite (small) Hopping?

\[ H_{RL} = \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j + \sum_i H_{Rabi}^i \]

- **Qualitative Phase Diagram in the J vs g plane:**
  - Small Hopping: Disordered Phase, Gapped, Insulating (but Not a Mott Insulator!)
    \[ \langle \sigma_i^x \rangle = 0 \quad \langle a_i \rangle = 0 \]
  - Large Hopping: Ordered Phase, Gapped, not a Superfluid!
    \[ \langle \sigma_i^x \rangle \neq 0 \quad \langle a_i \rangle \neq 0 \]

- **Z2 Parity Symmetry Breaking**
- **Quantum Phases and QPT are different from the physics of massive quantum particles on a lattice!**
Photon-Induced Qubit Interactions

- Photons always appear quadratically and can be “traced” out
  \[ \delta S_g = g \int_0^\beta d\tau \sum_i x_i(\tau) \sigma_i^x(\tau) \]

- Effective ferro-electric interaction between TLS
  \[ \delta S_{int} \sim \sum_{ij} \int d\tau \, d\tau' \, \sigma_i^x(\tau) \, K_{ij}(\tau - \tau') \, \sigma_j^x(\tau') \]

  - Kernel depends from photon band-structure
    \[ K_{ij}(\tau) = -\langle T_\tau x_i(\tau) \, x_j(\tau') \rangle_0 \]

  - Short-Ranged (space/time) unless gapless modes

- Qubit Frequency as a Transverse Field
Mean Field Phase Diagram

Mean Field Boundary Related to Static Susceptibility of Rabi model

\[
\frac{1}{zJ_c} = \int_0^\beta d\tau \langle T_\tau x(\tau) x(0) \rangle_{loc}
\]

“Ferro-electric” Insulator
\[
\langle \sigma_i^x \rangle \neq 0 \quad \langle a_i \rangle \neq 0
\]

“Para-electric” Insulator
\[
\langle \sigma_i^x \rangle = 0 \quad \langle a_i \rangle = 0
\]

Z2 Parity Symmetry Breaking

Negative detuning \( \delta \) favors the ordering phase

Universality Class??

\[ \delta = \omega_q - \omega_r \]
Universal Behavior at the QPT

From Rabi single site: low-energy doublet with a tiny splitting

- Opposite Parity: “Isospin”
  \[ \sum_x |\pm\rangle = \pm |\pm\rangle \]
- Rabi Hamiltonian in this subspace
  \[ H_{Rabi}^i \rightarrow \frac{\Delta}{2} \sum_i \]  

Project onto this low energy doublet:

\[
H_{eff} = \sum_{\langle ij \rangle} \left( J^x \sum_i \sum_j x + J^y \sum_i \sum_j y \right) + \frac{\Delta}{2} \sum_i \sum_i \]  

- Anisotropic XY model! Quantum Ising Universality Class
- Gapped phases, gapless at QCP -- Robust to weak dissipation!
The Role of Counter-Rotating Terms: From Jaynes-Cumming to Rabi
From Jaynes-Cumming to Rabi

\[ H_{Rabi} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g (a^\dagger \sigma^- + hc) + g' (a^\dagger \sigma^+ + hc) \]

Structure of the Ground State vs \( g' \)/\( g \)

\[ |\Psi\rangle = \sum c_n^\alpha |n \alpha\rangle \quad P_n^\alpha = |c_n^\alpha|^2 \]

Probabilities of having n polaritons

Level Crossings between different parity sectors at finite \( g' \)/\( g \)
The Fate of Mott Lobes

What happens to the Jaynes-Cumming Mott Lobes as we tune $g'/g$?

Lobes at fixed Parity survive at any finite $g'/g \neq 1$ but shift up.

### Graphs:

1. For $g = 0.5 \omega_0$, $\Pi = +1$ and $\langle a \rangle = 0$, $\langle \sigma^x \rangle = 0$.
2. For $g = 1.1 \omega_0$, $\Pi = -1$ and $\Pi = +1$.
3. For $g = 3.5 \omega_0$, $\Pi = -1$, $\Pi = -1$, and $\Pi = +1$.
4. For $g = 2.5 \omega_0$, $\Pi = +1$ and $\Pi = -1$.
The Role of Counter-Rotating Terms

Effective Action

\[ Z = \text{Tr} \ e^{-\beta H} = \int \prod_i D\Psi_i \ D\Psi_i^* \ e^{-S_{eff}[\Psi_i^*, \Psi_i]} \]

\[ S_{eff}[\Psi_i^*, \Psi_i] = \int_0^\beta d\tau \ \sum_{\langle ij \rangle} \Psi_i^* J_{ij}^{-1} \Psi_j + \sum_i \Gamma[\Psi_i^*, \Psi_i] \]

M. Fisher et al, PRB(1989) for Bose-Hubbard

\[ \Gamma[\Psi^*, \Psi] = \log \left\langle e^{-\int_0^\beta d\tau \left( \Psi^*(\tau) a(\tau) + a^\dagger(\tau) \Psi(\tau) \right)} \right\rangle \]

“Local” Physics (Rabi Non-Linearity)

Standard Field Theory of U(1) Superfluid-to-Mott Transition...

\[ S_{eff} = \int_0^\beta d\tau \left( K_1 \Psi_i^* \partial_\tau \Psi + K_2 |\partial_\tau \Psi|^2 + K_3 |\nabla \Psi|^2 + r |\Psi|^2 + u |\Psi|^4 \right) \]

.....+ Relevant Operators (explicit breaking down to \( \mathbb{Z}_2 \))

\[ S_{CRT} = \int_0^\beta d\tau \lambda_g' \left( \Psi \Psi + \Psi^* \Psi^* \right) \]

K. Damle & S. Sachdev, PRL(96)
Summing Up

- Rabi Hubbard Lattice Model
  - Use Counter-Rotating Terms to “polarize the vacuum”
  - Non trivial Ground State, no driving, no chemical potential
  - Relevant perturbation, drives the system toward a Z2 QCP

Perspectives:

- Effect of Driving and Dissipation
- Real Circuit QED architecture (flux qubits?)
- Effect of Disorder
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Basics of Superconducting Circuits (I)

- Transmission Line Resonator

\[ \mathcal{H}_r = \int_0^d dx \left( \frac{q^2(x)}{2c} + \frac{\phi^2(x)}{2l} \right) \]

Superconducting-based to give dissipation-less currents

- Circuit “Quantization”  

\[ q(x) = \sum_n A_n(x) \left( a_n + a_n^\dagger \right) \]

\[ \phi(x) = -i \sum_n B_n(x) \left( a_n - a_n^\dagger \right) \]

\[ \mathcal{H}_r = \sum_n \omega_n \ a_n^\dagger \ a_n \]

\[ \omega_n = \frac{\pi n}{d \sqrt{l c}} \]

Qubit: Mesoscopic SC system with “atom-like” spectrum

The Josephson Junction is the crucial (non-linear) ingredient!

Ex: The Cooper Pair Box

\[ \mathcal{H}_{CPB} = 4E_C (n - n_g)^2 - E_J \cos \varphi \]

Qubits come in different flavors (charge, flux, phase qubits)
Ultra-Strong Coupling of Light and Matter

How large can be the coupling between atom and photon in circuit QED?


Ultra-Strong Coupling Regime with Superconducting Circuits

Inductively Coupled Flux Qubit

\( g/\omega_r \approx 0.12 \)

Deviations from JC Physics

Many Other Platforms: Quantum Dots, 2DEG, WG Superlattices,..


Is the “standard” CQED picture working?


Polariton BEC in Semiconducting Micro-cavities

- Non-Equilibrium Superfluidity
- Effect of Pump and Dissipation on the BEC?
- Vortices, Defects, Hydrodynamics

Exp: J. Bloch, Yamamoto, D. Snoke...
Theory: J. Keeling, C. Carusotto, C. Ciuti..
Disordered Arrays of Coupled Cavities

\[
H = \sum_i \left( \omega_r + \delta_i \right) a_i^{\dagger} a_i + \sum_{ij} t_{ij} \left( a_i^{\dagger} a_j + h c \right)
\]

\[
t_{ij} = 2Z_0 C_{ij} (\omega_r + \delta_i) (\omega_r + \delta_j)
\]

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<th>$t/2\pi$ (MHz)</th>
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Quantum Impurity Physics with Light and Matter
Coupling a Microwave Resonator to Quantum Dot(s)+Leads


T. Frey et al PRL 107 256804 (2011) (ETH-Zurich, A. Wallraff Lab)

Similar setup in Jason Petta experiment (Princeton)
Kondo Effect in the phase of the microwave signal?

M. Delbeq et al PRL 107 256804 (2011)
(ENS-Paris, T. Kontos Lab)

- Relation between phase of transmitted photon and current flowing into the dot
- Kondo Physics(?) - Dot and Resonator Capacitively Coupled (charge...rather than spin)

- But: is Light really “quantum” here? Many (thermal) photons in the cavity..
- Coupling is global rather than local: AC Bias?
Phase Spectroscopy of a Resonator coupled to Dot+Leads

\[
H = \sum_{k,\alpha} \xi_{k\alpha} f_{k\sigma\alpha}^\dagger f_{k\sigma\alpha} + H_{loc}[c_{\sigma}^\dagger, c_{\sigma}] + \sum_{k\sigma\alpha} V_{k\alpha} (f_{k\sigma\alpha}^\dagger c_{\sigma} + c_{\sigma}^\dagger f_{k\sigma\alpha})
\]

\[
H_{loc} = \frac{U}{2} (n - 1)^2 + \omega_0 a^\dagger a + g x (n - 1)
\]
Phase Spectroscopy of a Resonator coupled to Dot+Leads

**Input-Output Relations**

\[ r(\omega) = \frac{\langle V_L^{\text{out}}(\omega) \rangle}{\langle V_L^{\text{in}}(\omega) \rangle} = 1 - 2i \, J_L(\omega) \, \chi_{xx}(\omega) \]

\[ \tan \varphi(\omega) = -\frac{2J_L(\omega) \, \chi_{xx}^R(\omega)}{1 - 2J_L(\omega) \, \chi_{xx}^I(\omega)} \]

**U=0 (Holstein Model)**

**Photon Propagator**
(knows about electrons)

**Inelastic Features as the bias hits the photon frequency** \( eV \sim \omega_0 \)
Non-Equilibrium Effective Theory for Photon in the Resonator

Can we integrate out the fermions?

\[ Z = \int D x_c D x_q \exp (iS_{eff} [x_c, x_q]) \]

Yes, perturbatively at large bias \( eV \gg \omega_0 \)

\[ S_{eff} = S_{loc}[x_c, x_q] + \frac{1}{2} \int dt \int dt' \sum_{\alpha=cl,q} x_\alpha(t) \Lambda_{\alpha\beta}(t, t') x_\beta(t') \]

Fermions induce anharmonicity, friction and noise for the photon
Effective Potential and Photon Anharmonicity

\[ \mathcal{S}_{loc}[x_c, x_q] = \int dt \ 2x_q \ (\ddot{x}_c + \omega_0^2 x_c + F(x_c)) \]

Anisotropy and Anharmonicity controlled by gate/bias voltages

\[ V_{eff}(x) = \frac{\omega_0^2 x^2}{2} + \int_0^x dx' \ F(x') \]

\[ V_{eff}(x) = \frac{\omega_*^2 x^2}{2} + \eta \ x^3 + g \ x^4 \]
Non-Equilibrium Fermionic Environment

At Finite Bias Noise and Friction do not satisfy Fluctuation-Dissipation Theorem...

\[ F(\Omega) = \frac{\Lambda^K(\Omega)}{\Lambda^R(\Omega) - \Lambda^A(\Omega)} \neq \coth \frac{\beta \Omega}{2} \]

.. but low-frequency modes are effectively thermal! \( T_{eff}(V) \)
Summing Up

- Biased Quantum Dot coupled to Resonator
  - Phase Spectroscopy sensitive to Photon
  - Green’s function

- Non-Equilibrium Effective Theory for the Photon: Anharmonicity/friction/noise and Effective Temp

Perspectives:

- Coulomb Blockade and Kondo Regime?
- Quantum-Classical Crossover with increasing Input Power?
Thanks!